Practice:

1. Write each of the following series using sigma notation.
   (a) \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \)
   (b) \( \cos \left( \frac{3\pi}{4} \right) + \cos \left( \frac{4\pi}{5} \right) + \cos \left( \frac{5\pi}{6} \right) + \cos \left( \frac{6\pi}{7} \right) + \ldots \)
   (c) \( 3 - 6 + 9 - 12 + 15 - 18 + \ldots \)

2. Suppose \( \sum_{n=1}^{50} a_n = 10.1 \), \( \sum_{n=1}^{500} a_n = 10.2 \), \( \sum_{n=1}^{5000} a_n = 10.3 \), and \( \sum_{n=1}^{50000} a_n = 10.4 \). Do you think \( \sum_{n=1}^{\infty} a_n \) converges or diverges? Why?

3. What is the difference between \( \sum_{i=1}^{\infty} a_i \) and \( \sum_{n=1}^{\infty} a_n \)?

4. Does \( \sum_{n=4}^{\infty} \frac{n-2}{6n} \) converge or diverge? How can you tell?

Assignment:

5. Write each of the following series using sigma notation.
   (a) \( \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{273} + \ldots \)
   (b) \( \sin \left( \frac{2\pi}{8} \right) - \sin \left( \frac{\pi}{9} \right) + \sin \left( \frac{0}{10} \right) - \sin \left( \frac{-\pi}{11} \right) + \sin \left( \frac{-2\pi}{12} \right) - \sin \left( \frac{-3\pi}{13} \right) + \ldots \)

6. Explain in your own words what it means for a series to converge to 5.

7. Does \( \sum_{n=36}^{\infty} \frac{4n^2 - 2n + 4}{3n^2 - 6} \) converge or diverge? How can you tell?

8. What is the difference between \( \sum_{i=1}^{\infty} \frac{1}{i^2} \) and \( \sum_{n=0}^{\infty} \frac{1}{(n+1)^2} \)?

Answers to Practice problems:

1. (a) \( \sum_{n=2}^{\infty} \frac{1}{2^n} \)
   (b) \( \sum_{n=3}^{\infty} \cos \left( \frac{n\pi}{n+1} \right) \)
   (c) \( \sum_{n=1}^{\infty} (-1)^n 3n \)

2. It likely diverges because the sequence of partial sums does not appear to be converging to any specific value.
3. No difference
4. This series diverges because the terms don’t approach zero (they approach 1/6) as $n \to \infty$. 